Journal of Computational and Theoretical Nanoscience
Vol. 12, 1–7, 2015

Interaction Due to Thermal Source in Micropolar Thermoelastic Diffusion Medium

Ibrahim A. Abbas[1,2,*], Rajneesh Kumar[3], and Sachin Kaushal[4]

[1] *Department of Mathematics, Faculty of Science and Arts-Khulais, King Abdulaziz University, Jeddah, Saudi Arabia*
[2] *Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt*
[3] *Department of Mathematics, Kurukshetra University, Kurukshetra*
[4] *Department of Mathematics, M. M University, Mullana (Ambala), Haryana, India*

In present study, the deformation in an micropolar thermoelastic diffusion medium due to thermal source by the use of finite element method (FEM) is investigated in the context of Lord-Shulman (L-S) theory of thermoelastiicty. A special type of sources have been taken to show the utility of the approach. The components of displacement, stress, microrotation, temperature change and mass concentration are computed numerically and depicted graphically to show the impact of micropolarity, diffusion and relaxation times. Some particular and specials cases are also deduced from present investigation.

Keywords: Micropolar, Diffusion, Relaxation Times, Finite Element Method.

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1. INTRODUCTION

Classical elasticity is inadequate to represent the behavior of material containing laminates and granular fibers as analysis of such materials requires incorporating the theory of oriented media, for this reason, micropolar theories are developed by Eringen[1,2] for elastic solids, fluid and further for non-local polar fields. Also Nowacki[3] developed a theory of micropolar coupled thermoelasticity. Later on, Touchert et al.[4] formulated the basic equations of linear theory of micropolar coupled thermoelasticity. Chandrasekharaiah[5] derived the theory of micropolar thermoelasticity in which heat flux is included among the constitutive variables. Boschi and Iesan[6] extended generalized theory of micropolar thermoelastic that permits the transmission of heat as thermal waves at finite speed.

Diffusion is important in many life processes and is of great interest due to its various applications in geophysics and industrial application. These days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from deposits. Thermoelastic diffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. Nowacki[7–10] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves.

Gawinecki and Szymaniec[11] proved a theorem about global existence of the solution for a non-linear parabolic thermoelastic diffusion problem. Sherief et al.[12] developed the theory of generalized thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation of waves. Sherief and Shaleh[13] discussed a half space problem in the theory of generalized thermoelastic diffusion with one relaxation time. Kumar et al.[14,15] discussed source problems in micropolar thermodiffusive medium. Miglani and Kaushal[16] studied propagation of transverse and microrotational waves in micropolar generalized thermodiffusion elastic medium. The finite element method is well addressed in the last century and dominant numerical method, which needs less computation in addition to their high accuracy in literature due to which it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. Othman and Abbas[17] studied the effect of rotation on plane waves at the free surface of a fibrereinforced thermoelastic half-space using the finite element method. Abbas[18] investigated ramp-type heating in a generalized thermoelastic half space with the help of finite element analysis. Abbas et al.[19] studied response of thermal source in a transversely isotropic thermoelastic half-space with mass diffusion by finite element method. Recently,[20–22] variants problems in waves are studied. Other forms are described for example in the Refs. [23–25]. The counterparts of our problem

*Author to whom correspondence should be addressed.

in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods.[26–40]

In present work, the Lord and Shulman theory of thermoelasticity is applied to study the thermal source with the help of finite element method. Furthermore, numerical results for the components of displacement, stresses, temperature distribution, concentration and chemical potential are represented graphically to show the impact of relaxation time.

2. BASIC EQUATIONS

Following Eringen,[1] Nowacki,[7] Lord and Shulman,[26] the governing equations for homogeneous isotropic micropolar generalized thermoelastic diffusion in absence of body forces, body couples, heat sources and diffusive mass sources are:

The constitutive relations,

$$t_{kl} = \lambda u_{r,r}\delta_{kl} + \mu(u_{k,l} + u_{l,k}) + K(u_{l,k} - \varepsilon_{klm}\phi_m) - \beta_1 T\delta_{kl} - \beta_2 C\delta_{kl} \tag{1}$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k} \tag{2}$$

$$P = -\beta_2 e_{kk} + bC - aT \tag{3}$$

Stress equations of motion,

$$(\lambda+\mu)u_{l,lk} + (\mu+K)u_{k,ll} + K\varepsilon_{klm}\phi_{m,l} - \beta_1 T_{,k} - \beta_2 C_{,k} = \rho\ddot{u}_k \tag{4}$$

Couple stress equations of motion,

$$(\alpha+\beta)\phi_{l,lk} + \gamma\phi_{k,ll} + K\varepsilon_{klm}u_{m,l} - 2K\phi_k = \rho j\ddot{\phi}_k \tag{5}$$

Equation of heat conduction,

$$\rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \beta_1 T_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial e_{kk}}{\partial t} + aT_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial t} = K^* T_{,ii} \tag{6}$$

Equation of mass diffusion,

$$D\beta_2 e_{kk,ii} + DaT_{,ii} + \left(1+\tau^0\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial t} - DbC_{,ii} = 0 \tag{7}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (i = 1, 2, 3)$$

$$\beta_1 = (3\lambda + 2\mu + K)\alpha_t, \quad \beta_2 = (3\lambda + 2\mu + K)\alpha_c$$

λ, μ-Lame's constants, α_t-coefficient of linear thermal expansion, α_c-coefficient of diffusion expansion, ρ-density, K^*-thermal conductivity, C_E-specific heat, t_{ij}-components of stress tensor, m_{ij}-components of couple stress tensor, e_{ij}-components of strain tensor, $e = e_{kk}$, δ_{ij}-kronecker delta, u_i-displacement components, ϕ_i-microrotational components, C-concentration, j-microrotation interia, K, α, β, γ, a, b-material constant, t-time, T-absolute temperature, T_0-temperature of medium in its natural state assumed to be such that $|T/T_0| < 1$, D-thermoelastic diffusion constant, P-chemical potential per unit mass.

3. FORMULATION AND SOLUTION OF THE PROBLEM

We consider a homogeneous, isotropic micropolar generalized thermodiffusion elastic solid in undeformed state at temperature T_0, which we designate as the medium $z \geq 0$ in rectangular cartesian co-ordinate $Oxyz$. We consider thermoelastic plane wave in xz-plane with wave front parallel to y-axis and all the field variables depend only on x, z and t. As the problem considered is two dimensional, therefore the displacement component $\vec{u}$ and microrotation component $\vec{\phi}$ can be written as

$$\vec{u} = (u, 0, w), \quad \vec{\phi} = (0, \phi_y, 0) \tag{8}$$

Using Eq. (8) in Eqs. (4)–(7), we obtain

$$(\lambda+\mu)\frac{\partial e_0}{\partial x} + (\mu+K)\nabla^2 u - K\frac{\partial\phi_y}{\partial z} - \beta_1\frac{\partial T}{\partial x} - \beta_2\frac{\partial C}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$(\lambda+\mu)\frac{\partial e_0}{\partial z} + (\mu+K)\nabla^2 w + K\frac{\partial\phi_y}{\partial x} - \beta_1\frac{\partial T}{\partial z} - \beta_2\frac{\partial C}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2} \tag{10}$$

$$\gamma\nabla^2\phi_y + K\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2K\phi_y = \rho j\frac{\partial^2\phi_y}{\partial t^2} \tag{11}$$

$$\rho C_E\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \beta_1 T_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial e_0}{\partial t} + aT_0\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial t} = K^*\nabla^2 T \tag{12}$$

$$D\beta_2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + Da\nabla^2 T + \left(1+\tau^0\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial t} - Db\nabla^2 C = 0 \tag{13}$$

The following dimensionless quantities are introduced:

$$\begin{gathered} x' = \frac{\omega_1}{c_1}x, \quad z' = \frac{\omega_1}{c_1}z, \quad u' = \frac{\rho c_1\omega_1}{\beta_1 T_0}u \\ t'_{zz} = \frac{t_{zz}}{\beta_1 T_0}, \quad t'_{zx} = \frac{t_{zx}}{\beta_1 T_0}, \quad w' = \frac{\rho c_1\omega_1}{\beta_1 T_0}w \\ C' = \frac{\beta_2}{\rho C_1^2}C, \quad T' = \frac{\beta_1}{\rho C_1^2}T, \quad \phi'_y = \frac{\rho c_1^2}{\beta_1 T_0}\phi_y \\ \tau^{o'} = \omega_1\tau^o, \quad m'_{ij} = \frac{\omega_1}{c_1\beta_1 T_0}m_{ij}, \quad \tau'_o = \omega_1\tau_o \\ t' = \omega_1 t \end{gathered} \tag{14}$$

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where

$$c_1^2 = \left(\frac{\lambda + 2\mu + K}{\rho}\right) \quad \text{and} \quad \omega_1 = \frac{\rho C_E c_1^2}{K^*}$$

Using dimensionless quantities defined by the Eq. (14) in Eqs. (9)–(13), we obtain (after suppressing the primes).

$$a_1 \frac{\partial e_0}{\partial x} + a_2 \nabla^2 u - a_3 \frac{\partial \phi_y}{\partial z} - a_4 \frac{\partial T}{\partial x} - a_4 \frac{\partial C}{\partial x} = \frac{\partial^2 u}{\partial t^2} \quad (15)$$

$$a_1 \frac{\partial e_0}{\partial z} + a_2 \nabla^2 w + a_3 \frac{\partial \phi_y}{\partial x} - a_4 \frac{\partial T}{\partial z} - a_4 \frac{\partial C}{\partial z} = \frac{\partial^2 w}{\partial t^2} \quad (16)$$

$$a_5 \nabla^2 \phi_y + a_6 \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - a_7 \phi_y = \frac{\partial^2 \phi_y}{\partial t^2} \quad (17)$$

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial t} + a_8 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial e_0}{\partial t} + a_9 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial C}{\partial t} = \nabla^2 T \quad (18)$$

$$a_{10} \nabla^2 e_0 + a_{11} \nabla^2 T + a_{12} \left(1 + \tau^0 \frac{\partial}{\partial t}\right) \frac{\partial C}{\partial t} - a_{13} \nabla^2 C = 0 \quad (19)$$

where

$$a_1 = \frac{(\lambda + \mu)}{\rho c_1^2}, \quad a_2 = \frac{(\mu + K)}{\rho c_1^2}, \quad a_3 = \frac{K}{\beta_1 T_0}$$

$$a_4 = \frac{\rho c_1^2}{\beta_1 T_0}, \quad a_5 = \frac{\gamma}{\rho j c_1^2}, \quad a_6 = \frac{K \beta_1 T_0}{j \rho^2 c_1^2 \omega_1^2}$$

$$a_7 = \frac{2K}{j \rho \omega_1^2}, \quad a_8 = \frac{\beta_1^3 T_0^2}{K^* \omega_1^2 \rho^2 c_1^2}, \quad a_9 = \frac{a \beta_1 T_0 c_1^2}{K^* \omega_1 \beta_2}$$

$$a_{10} = \frac{D \beta_1 \beta_2 T_0}{\rho c_1^4}, \quad a_{11} = \frac{D a \rho}{\beta_1}, \quad a_{12} = \frac{\rho c_1^2}{\beta_2 \omega_1}$$

$$a_{13} = \frac{D b \rho}{\beta_2}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \quad e_0 = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$$

From the Eqs. (1)–(3), with the help of (8) and (14) (after suppressing the primes), we get the expressions for stress components and chemical potential as

$$t_{xx} = \frac{\partial u}{\partial x} + h_1 \frac{\partial w}{\partial z} - a_4 T - a_4 C$$

$$t_{zz} = \frac{\partial w}{\partial z} + h_1 \frac{\partial u}{\partial x} - a_4 T - a_4 C$$

$$t_{xz} = h_3 \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z}\right) + h_2 \left(\frac{\partial w}{\partial x} + \phi_y\right)$$

$$m_{xy} = g_1 \frac{\partial \phi_y}{\partial x}$$

$$P = h_4 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + h_5 C - h_6 T$$

where

$$h_1 = \frac{\lambda}{\rho c_1^2}, \quad h_2 = \frac{K}{\rho c_1^2}, \quad h_3 = \frac{\mu}{\rho c_1^2}, \quad h_4 = \frac{-\beta_1 T_0}{\rho c_1^2}$$

$$h_5 = \frac{b \rho c_1^2}{\beta_2^2}, \quad h_6 = \frac{a \rho c_1^2}{\beta_1 \beta_2}, \quad g_1 = \frac{\gamma \omega_1^2}{\rho c_1^4}$$

4. INITIAL CONDITIONS

The above equations are solved subjected to initial conditions

$$u = w = \phi_y = T = C = 0, \quad \dot{u} = \dot{w} = \dot{\phi}_y = \dot{T} = \dot{C} = 0, \quad t = 0$$

5. BOUNDARY CONDITIONS

We assume that, on the boundary $x = 0$ the displacement u of the body does not depends on x, hence we have

$$u'(0, z, t) = 0$$

and the medium is subjected to a rough and rigid foundation enough to prevent the displacement w at any time and any point of z, then, we have

$$w(0, z, t) = 0$$

$$m_{xy}(0, z, t) = 0$$

$$p = 0$$

$$T = T_1 H(t) H(|z| - 2l)$$

where $H()$ is the Heaviside unit step function, T_1 is the constant temperature applied on the boundary respectively.

6. FINITE ELEMENT FORMULATION

In this section, the governing equations of homogeneous isotropic micropolar thermoelastic diffusion solid are summarized, followed by the corresponding finite element equations. In the finite element method, the displacement components u, w, microrotation components ϕ_y, temperature change T and mass concentration C are related to the corresponding nodal values by

$$u = \sum_{i=1}^{m} N_i u_i(t), \quad w = \sum_{i=1}^{m} N_i w_i(t), \quad \phi_y = \sum_{i=1}^{m} N_i \phi_{yi}(t)$$

$$T = \sum_{i=1}^{m} N_i T_i(t), \quad C = \sum_{i=1}^{m} N_i C_i(t)$$

where m denotes the number of nodes per element, N_i are the shape functions. The eight node isoparametric, quadrilateral element is used for displacement, microrotation, temperature and concentration calculations. The weighting functions and the shape functions coincide, thus,

$$\delta u = \sum_{i=1}^{m} N_i \delta u_i(t), \quad \delta w = \sum_{i=1}^{m} N_i \delta w_i(t)$$

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$$\delta\phi_y=\sum_{i=1}^{m}N_i\delta\phi_{yi}(t),\quad \delta T=\sum_{i=1}^{m}N_i\delta T_i(t)$$
$$\delta C=\sum_{i=1}^{m}N_i\delta C_i(t)$$

It should be noted that appropriate boundary conditions associated with the governing Eqs. (15)–(19) must be adopted in order to properly formulate a problem Boundary conditions are either essential (or geometric) or natural (or traction) types. Essential conditions are prescribed displacement components u, w, microrotation components ϕ_y, temperature change T and mass concentration C, while, the natural boundary conditions are prescribed tractions, heat flux, mass flux and couple stress which are expressed as

$$t_{xx}n_x+t_{zx}n_z=\bar{\tau}_x,\quad t_{xz}n_x+t_{zz}n_z=\bar{\tau}_{zz}$$
$$q_xn_x+q_zn_z=\bar{q},\quad \eta_xn_x+\eta_zn_z=\bar{\eta},\quad m_{xy}n_x=\bar{m}$$

where n_x, n_y and n_z are direction cosines of the outward unit normal vector at the boundary, $\bar{\tau}_x$, $\bar{\tau}_z$ are the given tractions values, $\bar{q}$ is the given surface heat flux, $\bar{\eta}$ is the given surface mass flux and $\bar{m}$is the given couple traction component. In the absence of body force, the governing equations are multiplied by weighting functions and then are integrate over the spatial domain Ω with the boundary Γ. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. Thus, the finite element equations corresponding to Eqs. (15)–(19) can be obtained as

$$\sum_{e=1}^{m}\left(\begin{bmatrix}M_{11}^e&0&0&0&0\\0&M_{22}^e&0&0&0\\0&0&0&M_{33}^e&0\\M_{41}^e&M_{42}^e&0&M_{44}^e&M_{45}^e\\0&0&0&0&M_{55}^e\end{bmatrix}\begin{bmatrix}\ddot{u}^e\\\ddot{w}^e\\\ddot{\phi}_y^e\\\ddot{T}^e\\\ddot{C}^e\end{bmatrix}+\begin{bmatrix}0&0&0&R_{14}^e&R_{15}^e\\0&0&0&R_{24}^e&R_{25}^e\\0&0&0&0&0\\R_{41}^e&R_{42}^e&0&R_{45}^e&R_{45}^e\\0&0&0&R_{54}^e&R_{55}^e\end{bmatrix}\begin{bmatrix}\dot{u}^e\\\dot{w}^e\\\dot{\phi}_y^e\\\dot{T}^e\\\dot{C}^e\end{bmatrix}\times\left[\begin{bmatrix}K_{11}^e&K_{12}^e&K_{13}^e&K_{14}^e&K_{15}^e\\K_{21}^e&K_{22}^e&K_{23}^e&K_{24}^e&K_{25}^e\\K_{31}^e&K_{32}^e&K_{33}^e&0&0\\0&0&0&K_{44}^e&0\\K_{51}^e&K_{52}^e&0&K_{54}^e&K_{55}^e\end{bmatrix}\begin{bmatrix}u^e\\w^e\\\phi_y^e\\T^e\\C^e\end{bmatrix}\right]\right) \tag{20}$$

where the coefficients in above equation are given below

$$M_{11}^e=M_{22}^e=M_{33}^e=\int_\Omega[N]^T[N]\,d\Omega$$
$$M_{41}^e=\int_\Omega[N]^T\left[\frac{\partial N}{\partial x}\right]d\Omega,\quad M_{42}^e=\int_\Omega[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$M_{44}^e=\int_\Omega\tau_0[N]^T[N]\,d\Omega,\quad M_{45}^e=\int_\Omega a_9\tau_0[N]^T[N]\,d\Omega$$
$$M_{55}^e=\int_\Omega a_{12}\tau^0[N]^T[N]\,d\Omega$$
$$R_{14}^e=-\int_\Omega a_4[N]^T\left[\frac{\partial N}{\partial x}\right]d\Omega,\quad R_{15}^e=-\int_\Omega a_4[N]^T\left[\frac{\partial N}{\partial x}\right]d\Omega$$
$$R_{24}^e=-\int_\Omega a_4[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega,\quad R_{25}^e=-\int_\Omega a_4[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$R_{41}^e=\int_\Omega a_8\tau_0[N]^T\left[\frac{\partial N}{\partial x}\right]d\Omega,\quad R_{42}^e=\int_\Omega a_8[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$R_{44}^e=\int_\Omega[N]^T[N]\,d\Omega,\quad R_{45}^e=\int_\Omega a_9[N]^T[N]\,d\Omega$$
$$R_{54}^e=\int_\Omega a_{11}\left(\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\right)d\Omega$$
$$R_{55}^e=\int_\Omega a_{12}[N]^T[N]-a_{13}\left(\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\right)d\Omega$$
$$K_{11}^e=\int_\Omega(a_1+a_2)\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+a_2\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$K_{12}^e=\int_\Omega a_1\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$K_{13}^e=-\int_\Omega a_3[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$K_{14}^e=K_{15}^e=-\int_\Omega a_4[N]^T\left[\frac{\partial N}{\partial x}\right]d\Omega$$
$$K_{21}^e=\int_\Omega a_1\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial x}\right]d\Omega$$
$$K_{22}^e=\int_\Omega(a_1+a_2)\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]+a_2\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]d\Omega$$
$$K_{23}^e=-\int_\Omega a_3[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$K_{24}^e=K_{25}^e-\int_\Omega a_4[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$K_{31}^e=\int_\Omega a_6[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega,\quad K_{32}^e=-\int_\Omega a_6[N]^T\left[\frac{\partial N}{\partial z}\right]d\Omega$$
$$K_{33}^e=\int_\Omega a_5\left(\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\right)-a_7\,d\Omega$$
$$K_{44}^e=\int_\Omega K\left(\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\right)d\Omega$$

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$$K_{51}^{e}=\int_{\Omega} a_{10}[N]^{T}\left[\frac{\partial N}{\partial x}\right]\left(\left[\frac{\partial N}{\partial x}\right]^{T}\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^{T}\left[\frac{\partial N}{\partial z}\right]\right) d\Omega$$

$$K_{52}^{e}=\int_{\Omega} a_{10}[N]^{T}\left[\frac{\partial N}{\partial z}\right]\left(\left[\frac{\partial N}{\partial x}\right]^{T}\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^{T}\left[\frac{\partial N}{\partial z}\right]\right) d\Omega$$

$$K_{54}^{e}=\int_{\Omega} a_{10}\left(\left[\frac{\partial N}{\partial x}\right]^{T}\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^{T}\left[\frac{\partial N}{\partial z}\right]\right) d\Omega$$

$$K_{55}^{e}=-\int_{\Omega} a_{13}\left(\left[\frac{\partial N}{\partial x}\right]^{T}\left[\frac{\partial N}{\partial x}\right]+\left[\frac{\partial N}{\partial z}\right]^{T}\left[\frac{\partial N}{\partial z}\right]\right) d\Omega$$

Symbolically, the discretized equations of the Eq. (20) can be written as

$$Md+Rd+Kd=F^{\text{ext}}$$

where M, R, K and F^{ext} represents the mass, damping, stiffness matrices and external force vector, respectively; $d=[u\ w\ \phi_y\ T\ C]^T$. On the other hand, the time derivatives of the unknown variables have to be determined by newmark time intergration method or other methods (see Ref. [41]).

7. NUMERICAL RESULTS AND DISCUSSION

For numerical computations, the values of relevant parameters for micropolar thermoelastic diffusion with relaxation times are taken as under micropolar elastic parameters:

$$\lambda=9.4\times10^{10}\ \text{N m}^{-2},\quad \mu=4.0\times10^{10}\ \text{N m}^{-2}$$

$$\text{K}=1.0\times10^{10}\ \text{N m}^{-2},\quad \gamma=0.779\times10^{-9}\ \text{N}$$

$$\rho=1.74\times10^{3}\ \text{Kg m}^{-3}$$

Thermoelastic diffusion parameters:

$$C^{*}=1.0\ \text{J kg}^{-1}\ \text{deg}^{-1},\quad K^{*}=1.7\times10^{2}\ \text{J m}^{-1}\ \text{sec}^{-1}\ \text{deg}^{-1}$$

$$\alpha_t=1.78\times10^{-5}\ \text{K}^{-1},\quad \alpha_c=1.98\times10^{-4}\ \text{m}^3\ \text{kg}^{-1}$$

$$b=0.9\times10^{6}\ \text{m}^5\ \text{kg}^{-1}\ \text{s}^{-2},\quad D=0.85\times10^{-8}\ \text{kg s m}^{-3}$$

$$a=1.2\times10^{4}\ \text{m}^2\ \text{s}^{-2}\ \text{K}^{-1},\quad j=0.2\times10^{-19}\ \text{m}^2$$

$$T_0=298\ \text{K},\quad \tau_0=0.02\ \text{s},\quad \tau^0=0.01\ \text{s}$$

The variations of variation of compontents of displacement, stresses, concentration, temperature and chemical potential with distance x is shown graphially for L-S and CT theory of thermoelasticity. The solid line, small dashed line corresponds to CT theory for $z=0.2$ and $z=0.3$, whereas dashed line (Bold) and dashed line represents the cases of L-S theory for $z=0.2$ and $z=0.3$.

It is noticed from Figure 1 that initially values of u for L-S and CT at $z=0.3$ are greater in magnitude as compared to those observed at $z=0.2$ and as x increases, values of u approaches towards origin.

Figure 2 shows the variation of w with distance x. It is noticed that the values of w for L-S and CT theory shows non-uniform pattern in first half of interval and further as x increases value of w shows steady state about origin.

It is noticed from Figure 3, which is a plot of ϕ_y that initially value of ϕ_y for L-S and CT theory at $z=0.3$ decreases with greater magnitude as compared to those observed at $z=0.2$ and as x increases, value of ϕ_y shows small variations about zero value.

It is observed from Figure 4, which is a plot of C that value of C decreases in the entire range and approaches towards zero value.

It is observed from Figure 5 that values of T for both L-S and CT theory of thermoelasticity decreases in first half of interval and thereafter approaches to zero value.

From Figure 6, It is noticed that value of t_{xx} increases in the entire range except in the range $0.1\le x\le0.3$ where decreasing trends are noticed.

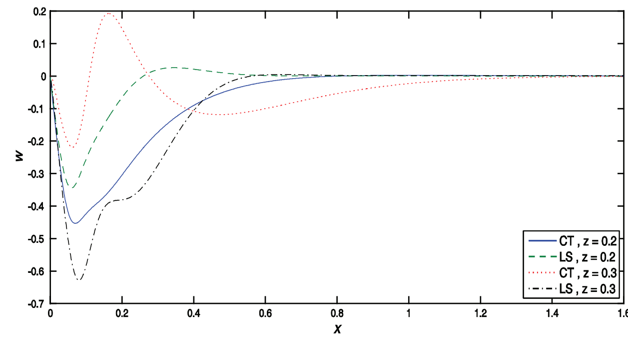

Fig. 2. Normal displacement distribution.

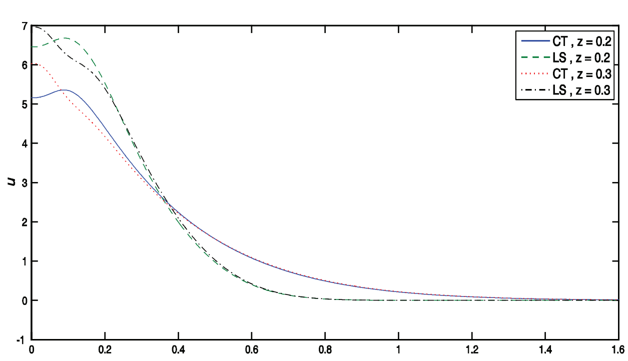

Fig. 1. Tangential displacement distribution.

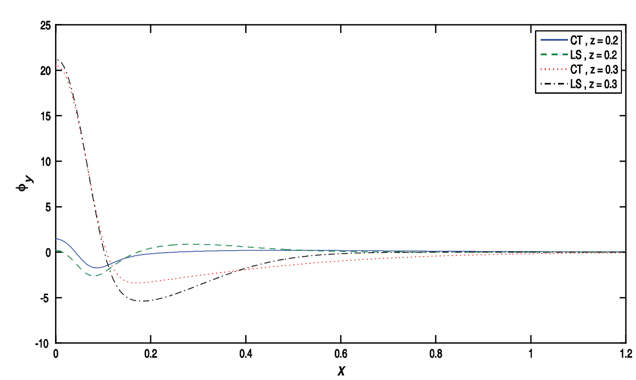

Fig. 3. Microrotation distribution.

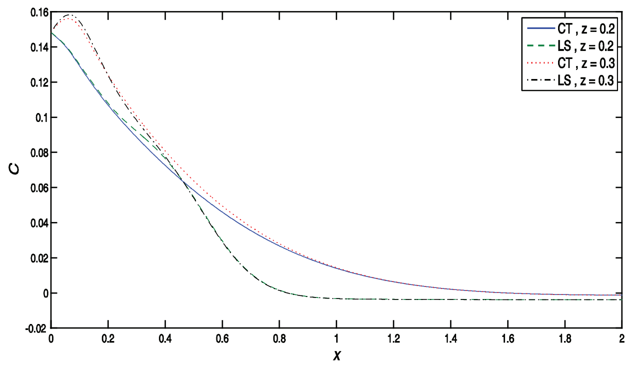

Fig. 4. Mass concentration distribution.

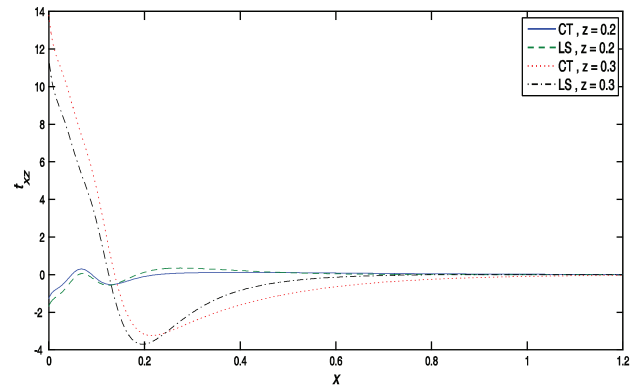

Fig. 7. Tangential stress t_{xz} distribution.

It is observed from Figure 7, value of t_{zx} for CT theory are greater in magnitude as compared to those obtained for L-S theory at $z = 0.3$ and $z = 0.2$.

It is noticed form Figure 8, which is plot for t_{zz} that the values of t_{zz} at $z = 0.3$ for L-S and CT theory decreases in the range $0.1 \leq x \leq 0.3$ and increases in the rest of interval. Whereas, value of t_{zz} at $z = 0.2$ for both theories increases in first half of interval and then it shows steady state about zero value.

Figure 9 shows the variations of m_{xy} with distance x. It is noticed that values of m_{xy} for L-S and CT theory decreases initially, magnitude of values at of values at $z = 0.3$ are greater in comparison to those observed at $z = 0.2$ and after that till $x = 0.3$, values of m_{xy} increases and as increases, value of m_{xy} shows small variations about zero value.

It is noticed that from Figure 10, that values of P decreases in entire range for both theories of thermoelasticity, magnitude of values for L-S theory are greater in comparison to those observed for CT theory, which reveals the impact of relaxation time.

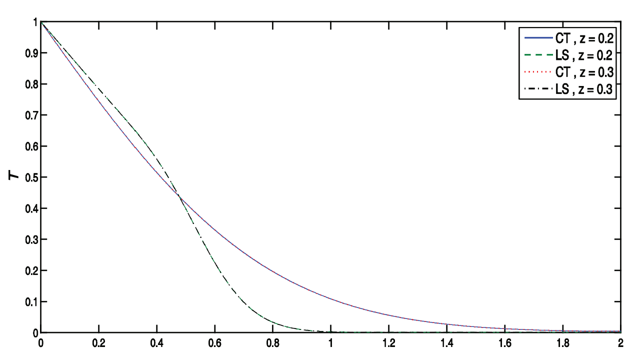

Fig. 5. Temperature distribution.

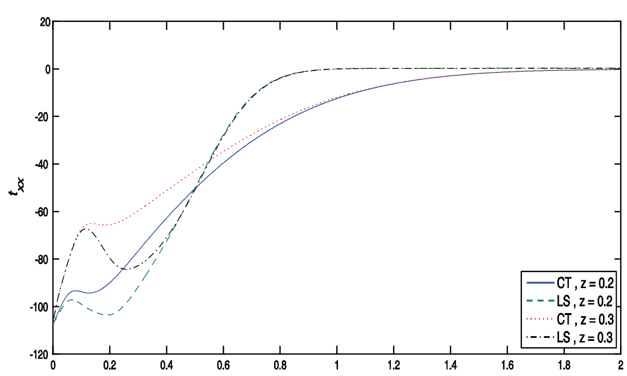

Fig. 6. Normal stress t_{xx} distribution.

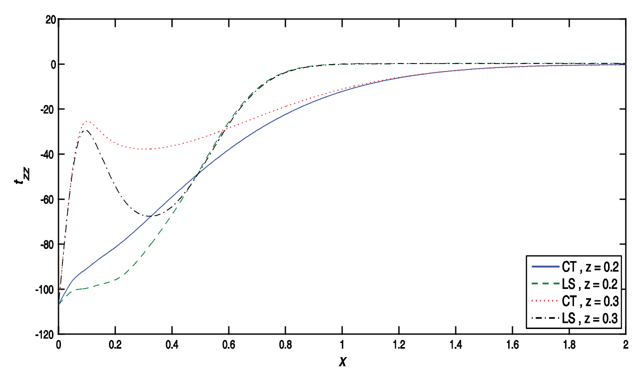

Fig. 8. Normal stress t_{zz} distribution.

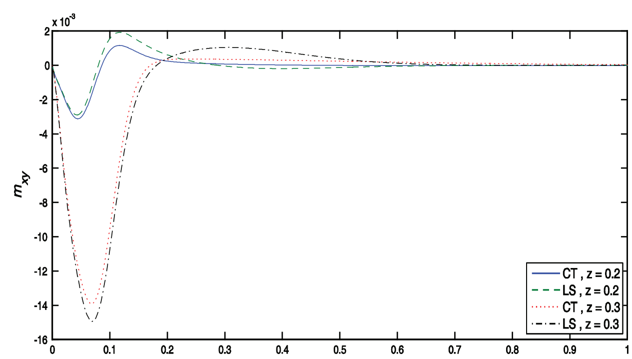

Fig. 9. Couple stress distribution.

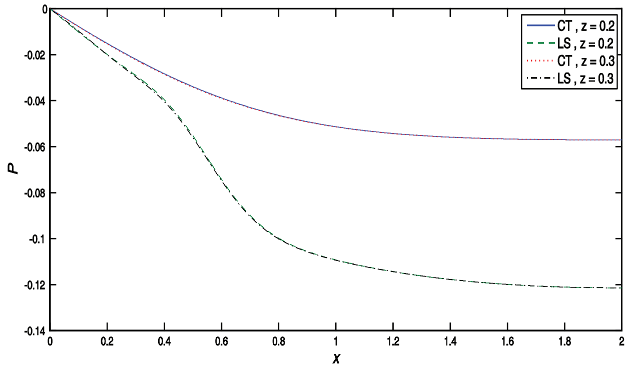

Fig. 10. Chemical potential distribution.

8. CONCLUSION

A two dimensional problem in an homogenous isotropic micropolar thermodiffusion elastic medium is studied in the context of the Lord-Shulman theory of thermoelasticity. The problem has been solved numerically by using the finite element method. It is observed from the above numerical discussion that near the point of application of source the impact of both theories of thermoelasticity has significant effect on all field quantities and as x increases the values of various components so obtained, tends to zero value in an oscillatory manner. It is also noticed different values of z shows impact on components so obtained.

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Received: 17 April 2014. Accepted: 14 May 2014.

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